even answers: p. 506

24) $\frac{2}{27}(2+3x)^{1/2}(3x-4) + C$

36) $\frac{\pi}{4} - \frac{1}{2}$

38) $e - 2$

62) $A = -\frac{2}{e} + 1$

36) $\int_0^1 x \arcsin x^2\,dx$ $\quad u = \arcsin x^2 \quad dv = x\,dx$

$du = \frac{2x}{\sqrt{1-x^4}}\,dx \quad v = \frac{x^2}{2}$

24) $\int \frac{x}{\sqrt{2+3x}}\,dx \quad u = x \quad dv = (2+3x)^{-1/2}dx$

$du = dx \quad v = \frac{2}{3}(2+3x)^{1/2}$

$= \frac{2}{3}x(2+3x)^{1/2} - \frac{1}{3}\cdot\frac{2}{3}\int 3(2+3x)^{1/2}dx$

$= \frac{2}{3}x(2+3x)^{1/2} - \frac{2}{9}\cdot\frac{2(2+3x)^{3/2}}{3} + C$

$\frac{2}{3}x(2+3x)^{1/2} - \frac{4}{27}(2+3x)^{3/2} + C$

$\frac{2}{3}(2+3x)^{1/2}\left(x - \frac{2}{9}(2+3x)\right) + C$

62)

$$A = \frac{1}{9}\int_0^3 x e^{-x/3}\, dx$$

Individual Quiz # 2 Tuesday, June 5

Euler's Method

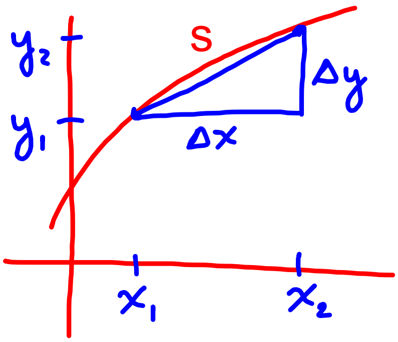
Integration by parts

Separation of variables to find a particular solution to a differential equation.

Arc Length

Let s = Arc Length

We can estimate the length of a curve by considering line segments and the distance formula, making those line segments smaller and smaller, then summing them up!

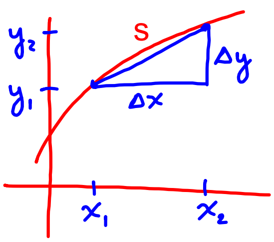


$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

Let s = Arc Length

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$



Approximating the length of the curve by summing up very small line segments:

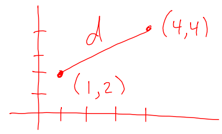
$$\sum \frac{\sqrt{(\Delta x)^2 + (\Delta y)^2}}{\sqrt{(\Delta x)^2}} \Delta x$$

$$\sum \sqrt{\frac{(\Delta x)^2}{(\Delta x)^2} + \frac{(\Delta y)^2}{(\Delta x)^2}} \, \Delta x$$

$$\lim_{n \to \infty} \sum_{i=1}^{n} \sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2} \, \Delta x$$

$$s = \int_a^b \sqrt{1 + [f'(x)]^2} \, dx$$

Test it:



$$d = \sqrt{(4-1)^2 + (4-2)^2}$$
$$= \sqrt{9+4}$$
$$= \sqrt{13}$$

$$S = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx \rightarrow f'(x) = m = \frac{2}{3}$$

$$= \int_1^4 \sqrt{1 + \left(\frac{2}{3}\right)^2}\, dx$$

$$\int_1^4 \sqrt{\frac{13}{9}}\, dx$$

$$\frac{1}{3}\int_1^4 \sqrt{13}\, dx$$

$$\frac{1}{3}\Big[\sqrt{13}\, x\Big]_1^4$$

$$\frac{1}{3}\left(4\sqrt{13} - \sqrt{13}\right)$$

$$\frac{1}{3}\left(3\sqrt{13}\right) = \sqrt{13}$$

You try: Find the length of the curve by hand.

$$y = \frac{4\sqrt{2}}{3}x^{3/2} - 1 \quad \text{on } [0,1]$$

$$y' = 2\sqrt{2x}$$

$$S = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

$$S = \int_0^1 \left(1 + \left(2\sqrt{2x}\right)^2\right)^{1/2} dx$$

$$\frac{1}{8}\int_0^1 8(1+8x)^{1/2} dx$$

$$u = 1 + 8x$$
$$du = 8\, dx$$

for $x = 0 \rightarrow u = 1$

$x = 1 \rightarrow u = 9$

$$= \frac{1}{8}\left[\frac{2u^{3/2}}{3}\right]_1^9$$

$$= \frac{1}{8}\cdot\frac{2}{3}\left[u^{3/2}\right]_1^9$$

$$= \frac{1}{12}(27 - 1)$$

$$= \boxed{\frac{13}{6}}$$

Another: Find the length of the curve

$f(x) = \frac{x^3}{6} + \frac{1}{2x}$ on $\left[\frac{1}{2}, 2\right]$

$f'(x) = \frac{1}{2}\left(x^2 - \frac{1}{x^2}\right)$ $\qquad 2x^2\left(-\frac{1}{x^2}\right)$

$$\int_{1/2}^{2} \sqrt{1 + \left(\frac{1}{2}\left(x^2 - \frac{1}{x^2}\right)^2\right)}$$

$$\int \sqrt{\frac{4}{4} + \frac{1}{4}\left(x^4 - 2 + \frac{1}{x^4}\right)}\, dx$$

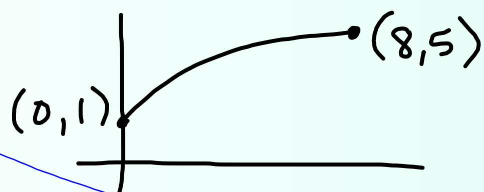
$$\frac{1}{2}\int_{1/2}^{2} \sqrt{\left(4 + x^4 - 2 + \frac{1}{x^4}\right)}\, dx$$

$$\frac{1}{2}\int_{1/2}^{2} \sqrt{\left(x^4 + 2 + \frac{1}{x^4}\right)}\, dx$$

$$\frac{1}{2}\int_{1/2}^{2} \sqrt{\left(x^2 + \frac{1}{x^2}\right)^2}\, dx$$

You can also find the length of the curve with respect to y:

$x = (y-1)^{3/2}$



$$\int_{1}^{5} \sqrt{1 + (\qquad)^2}\, dy \qquad \frac{3}{2}(y-1)^{1/2}(1)$$

HW: 6.4 practice, Arc Length

p. 327, # 4 - 7 all, 10 - 14 even

even answers follow...

<u>Week 10 Classwork</u>

2 Green WS's

due turned in tomorrow

4) $\frac{8}{27}(10\sqrt{10} - 1)$

6) $5\sqrt{5} - 2\sqrt{2}$

10) $s = \int_0^1 \frac{\sqrt{(x+1)^4 + 1}}{(x+1)^2}\,dx$

12) $s = \int_{-8}^{8} \frac{\sqrt{9x^{4/3} + 1}}{3x^{2/3}}\,dx$

14) $s = \int_0^{a/2} \frac{a}{\sqrt{a^2 - y^2}}\,dy$